Measures of Population Change

We have been discussing how we estimate parameters needed to further our understanding of the dynamics of animal populations.

Now we are shifting gears to begin the process of using the parameter estimates we generate from marked animals to model the dynamics of these populations.
To prepare the way, we first need to think about how we measure population change.

After all, our principal interest is in the direction and magnitude of change in wild populations.
\[ N_{t+1} = N_t + B_t - D_t + I_t - E_t \]
For continuous population change we use the parameter $r$ to assess that rate of change. For example:

$$N(t) = N(0)e^{rt}$$

$r$ is the intrinsic rate of population growth.
For the discrete time case, such as the population model a couple of slides back, we can write the expression:

\[ N_{t+1} = RN_t + N_t \]

\[ N_{t+1} = (R + 1) \cdot N_t \]

where \( R \) is the net discrete (or geometric) per capita rate of increase. Note if \( t \) is generation time (usually denoted \( T \)), \( R \) is the number of offspring recruited into the breeding population in an individual’s lifetime.
We can also write relationships between \( N_{t+1} \) and \( N_t \) as follows:

\[
N_{t+1} = \lambda N_t
\]

where \( \lambda \) is the discrete (or geometric) per capita rate of growth (definitions from Case 2000).
For projection multiple time steps into the future we can write the following:

\[ N_{t+2} = \lambda N_{t+1} \]
\[ N_{t+1} = \lambda N_t \]
\[ \Rightarrow \]
\[ N_{t+2} = \lambda (\lambda N_t) = \lambda^2 N_t \]

In general:

\[ N_{t+T} = \lambda^T N_t \]
\[ \lambda = e^r \]

\[ \Rightarrow \]

\[ \ln \lambda = r \]

Relating continuous to discreet time.
When a population is in a stable age distribution we can calculate $\lambda$ from the Euler-Lotka equation.

$$1 = \sum_{x=1}^{\max} l_x m_x \lambda^{-x}$$

Thus, demographic parameters of fecundity and probability of surviving to age $x$ determine $\lambda$. 
We will see in the next couple of weeks that we can calculate $\lambda$ from the fecundities and survival probabilities we enter into a population projection matrix. (We call such a matrix a Leslie matrix after the man who discovered it). These matrices will provide the principal way we model population dynamics in NRES 488/688.
As we will see next lecture, the Leslie matrix projects each age class from one time step to the next.

In the Leslie matrix, itself, we assume parameters (e.g., survival) are constant. As such, these matrices do not allow us to directly model biological processes such as density dependence.
Nevertheless, the Leslie matrix is a powerful tool for assessing whether a particular combination of fecundities and survival probabilities produce an increasing or decreasing population.

We can, thus, examine the impact of management actions on population dynamics by changing the elements of the matrix to mimic the expected response to management.

For example, a hunting closure might be expected to increase survival (if harvest is additive to other sources of mortality) and we could examine the implications of such an action using a population model.
When $\lambda$ is estimated from a Leslie matrix we call $\lambda$ the dominant eigenvalue of the matrix. (More about this later.)

The stable age distribution is the right eigenvector of the matrix associated with $\lambda$.

A population with constant birth and death rates (usually unrealistic) will always reach a stable age distribution.
If there is random variation in births and deaths, the population will not reach a stable age distribution.

Note, in such circumstances we can empirically estimate $\lambda$ from the following (if we know population size):

$$\lambda = \frac{N_{t+1}}{N_t}$$
It is also possible to estimate $\lambda$ directly from capture-recapture data. Roger Pradel developed this method, whereby survival (and emigration) is estimated from a standard capture history, while recruitment (and immigration) is estimated by when individuals appear in the capture history.

$$\lambda = f + \phi$$

Turning the capture around backwards, appearances become disappearances and appearance (recruitment) is estimated like it was a mortality.