STOCHASTICITY IN VITAL RATES

So far, we have considered population models in which we assumed that birth and death rates were constant and known with certainty.

In fact,

(1) we can’t know birth and death rates with certainty because we rely on sampling and estimation to understand vital rates;

(2) vital rates may vary spatially, temporally, and among individuals.
Case (2000) does a very nice job of talking about spatial patterns.

We’re going to ignore spatial variation, not because it’s unimportant, but in the interest of time. Spatial variation is every bit as important as temporal variation and is a fundamental determinant of population dynamics.

Today we’re going to focus on temporal and individual variation.
Effect of Stochasticity and the Frequency of Good and Bad Years for Nest Success
VARIATION IN $\lambda$ AND POPULATION INCREASE

$\lambda = 1.5$

- **CONSTANT**
- **VARIABLE**
Because population increase is a multiplicative process, the geometric mean of \( \lambda \) is a more appropriate measure of growth rate than is the arithmetic mean.

\[
\text{geometric mean} = \sqrt[n]{\lambda_1 \lambda_2 \lambda_3 \cdots \lambda_n}
\]
The geometric mean is always less than or equal to the arithmetic mean.

$$\ln(\text{geo mean}) = \ln[\prod \lambda_i]^{1/n}$$

$$= (1/n)\ln(\lambda_1 \lambda_2 \lambda_3 \lambda_4 \ldots \lambda_n)$$

$$= (1/n)[\ln \lambda_1 + \ln \lambda_2 + \ln \lambda_3 \ldots \ln \lambda_n]$$

$$= (1/n)(n)(\text{arithmetic mean ln}\lambda)$$

$$= \text{arithmetic mean ln}\lambda$$
Figure 2.21
Geometric growth when $\lambda$ each year is drawn randomly from a uniform distribution [0.1 and 2.1]. Compare to Figures 2.19 and 2.20. These populations tend to decline because the geometric mean $\lambda$ is less than 1.
It is important to keep in mind that in nature we observe one of the many possible population trajectories.

From Case (2000)
Figure 2.22
Each normal curve gives the probability density function for $(\ln \lambda)_T$ (the mean of the $\ln \lambda_i$ after $T$ years); $T = 1, 5, \text{ and } 20$. The $\ln$ of the geometric mean $\lambda$ for these years is equal to this mean. In this example, $\ln \lambda_i$ for each year is drawn from a universe distribution with a negative mean: $u_G = -0.5$ and a standard deviation of 1. Hence the average $\lambda$ is less than 1 ($e^{-0.5} = 0.6065$). The shaded regions represent the probability that $(\ln \lambda)_T$ is greater than 0, which corresponds to a population that has increased in size after $T$ years. As time goes on, this probability decreases. When the geometric mean $\lambda$ is less than 1, the typical population declines to extinction as $T$ increases.

From Case (2000)
Figure 2.23
An example of a situation where the mean population size increases geometrically, but most populations decline to extinction. Here $\lambda$ is drawn randomly from a uniform distribution [0.1 to 2.1]. The red lines show 100 replicate populations beginning from $N_0 = 2$. The black line marks a population size of one individual.

Realized mean $N$ at end: 277.4
Expected $N$ at end: $(2)(1.1)^{50} = 234.7$

Geometric mean $\lambda \approx 0.9$
Arithmetic mean = 1.1

From Case (2000)
When $\lambda$ varies among individuals, stochasticity has less effect as the population increases because of the law of large numbers.

From Case (2000)